Delineating Efficient Portfolios

描述有效的组合

Expected return and volatility of a two-asset portfolio

- The Expected Return of a two-asset portfolio

$$E(R_P) = w_1 E(R_1) + w_2 E(R_2)$$

- The Variance of a two-asset portfolio

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov_{1,2}$$

  - Or

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$$

Expected return and volatility of a two-asset portfolio

Example: Expected return and volatility for a two-asset portfolio

Using the information in the following figure, calculate the expected return and standard deviation of the two-asset portfolio.

Characteristics for a Two-Stock Portfolio

	Caffeine Plus	*Sparklin'*
Amount invested	$40,000	$60,000
Expected return	11%	25%
Standard deviation	15%	20%
Correlation	0.30	

Answer:

First, determine the weight of each stock relative to the entire portfolio. Since the investments are $40,000 and $60,000, we know the total value of the portfolio is $100,000:

w_c = investment/portfolio value = $40,000 / $100,000 = 0.40

w_s = investment/portfolio value = $60,000 / $100,000 = 0.60

Expected return and volatility of a two-asset portfolio

Next, we determine the expected return on the portfolio:

$$E(R_p) = w_c E(R_c) + w_s E(R_s)$$

$$E(R_p) = (0.40)(0.11) + (0.60)(0.25) = 0.1940 = 19.40\%$$

Then, we calculate the variance of the portfolio:

$$\begin{aligned}\sigma_p^2 &= w_c^2\sigma_c^2 + w_s^2\sigma_s^2 + 2w_c w_s \rho_{cs}\sigma_c\sigma_s \\ &= (0.40)^2(0.15)^2 + (0.60)^2(0.20)^2 + 2(0.40)(0.60)(0.30)(0.15)(0.20) \\ &= 0.02232\end{aligned}$$

And, finally, the standard deviation of the portfolio:

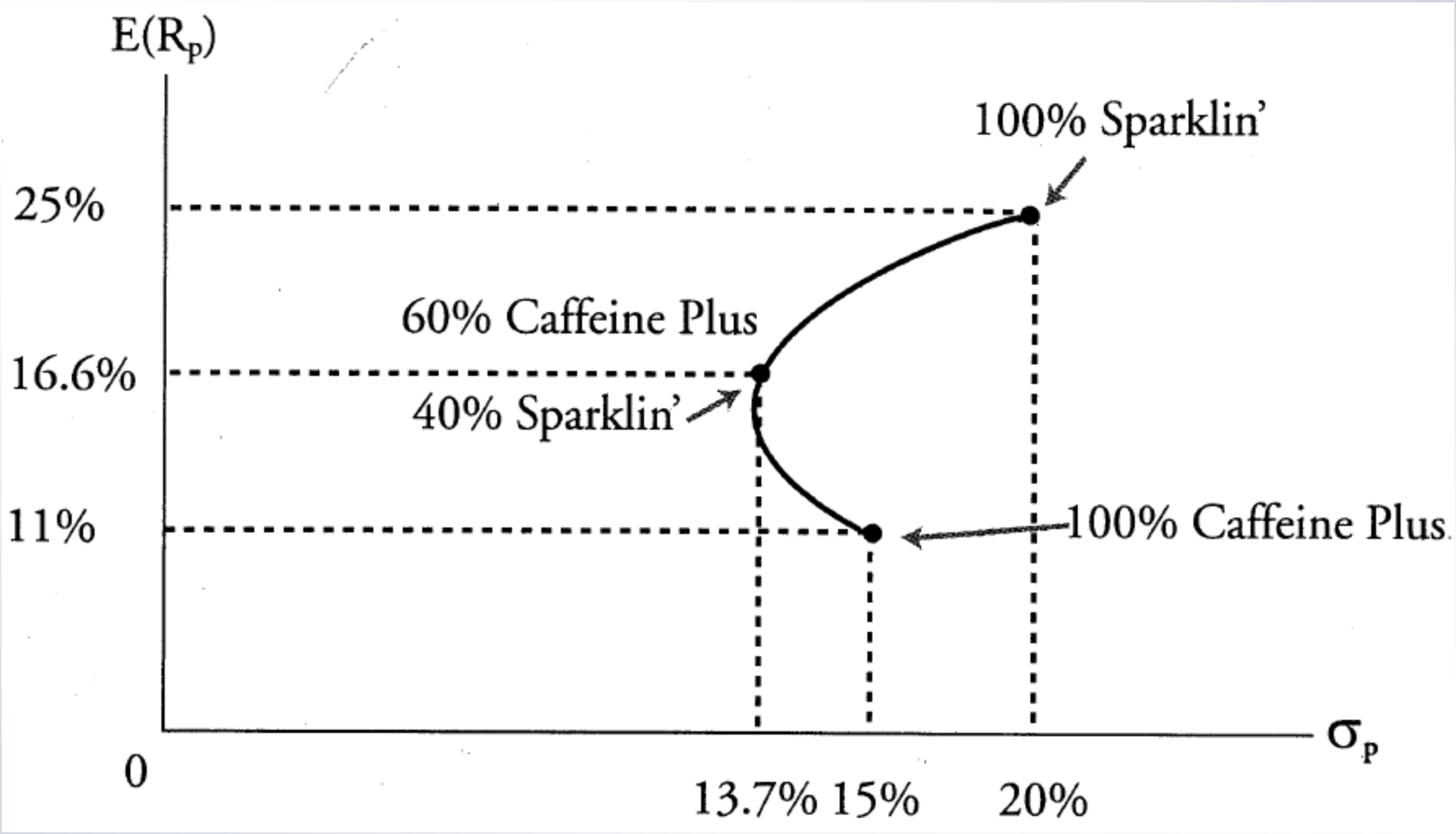
$$\sigma_p = \sqrt{\sigma_p^2} = \sqrt{0.02232} = 0.1494 = 14.94\%$$

The Portfolio Possibilities Curve

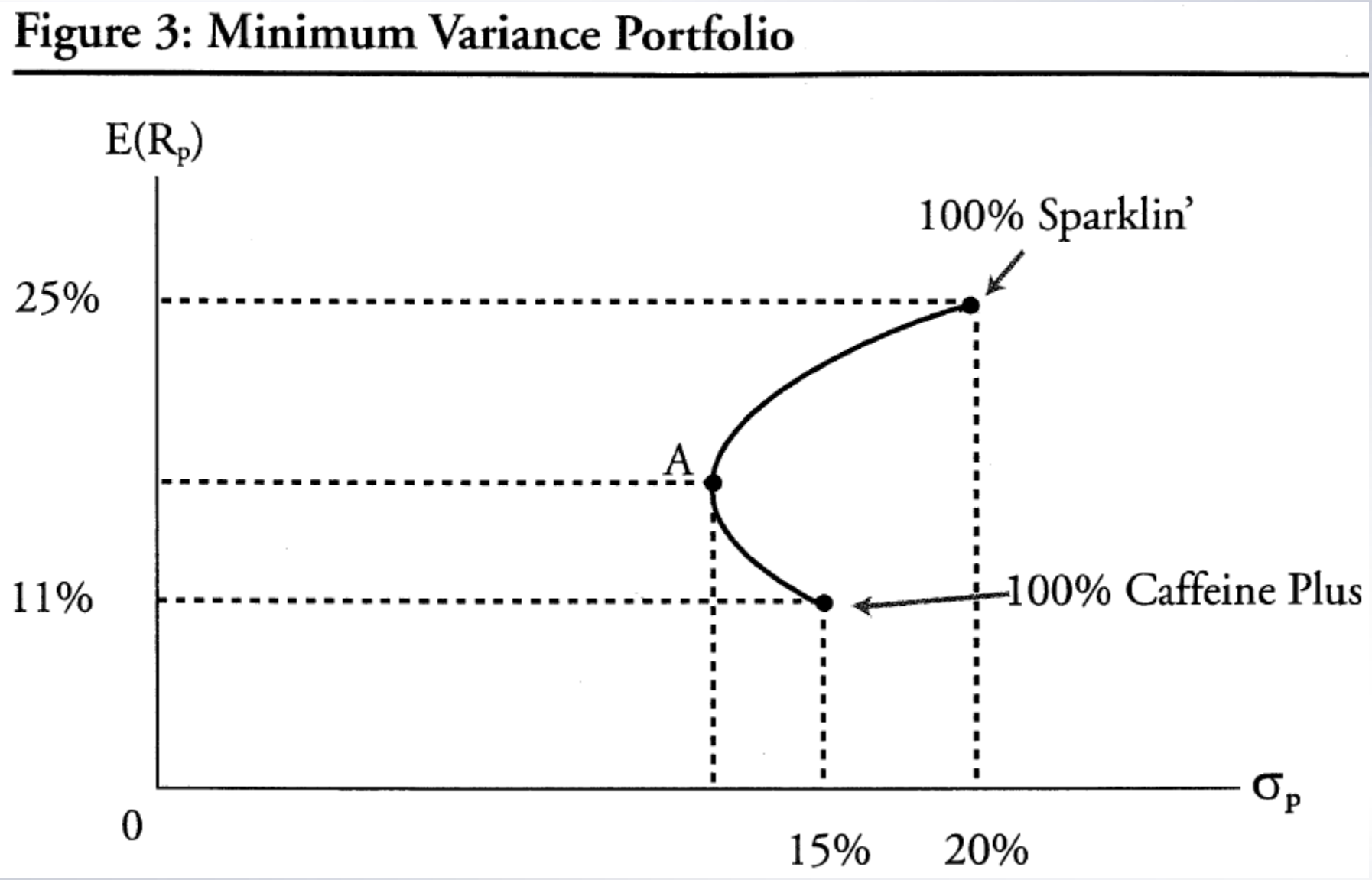
- Portfolio Returns for Various Weights of Two Assets

$W_{\text{Caffeine Plus}}$	100%	80%	60%	40%	20%	0%
W_{Sparklin}	0%	20%	40%	60%	80%	100%
$E(R_p)$	11.00%	13.80%	16.60%	19.40%	22.20%	25.00%
σ_p	15.00%	13.74%	13.72%	14.94%	17.10%	20.00%

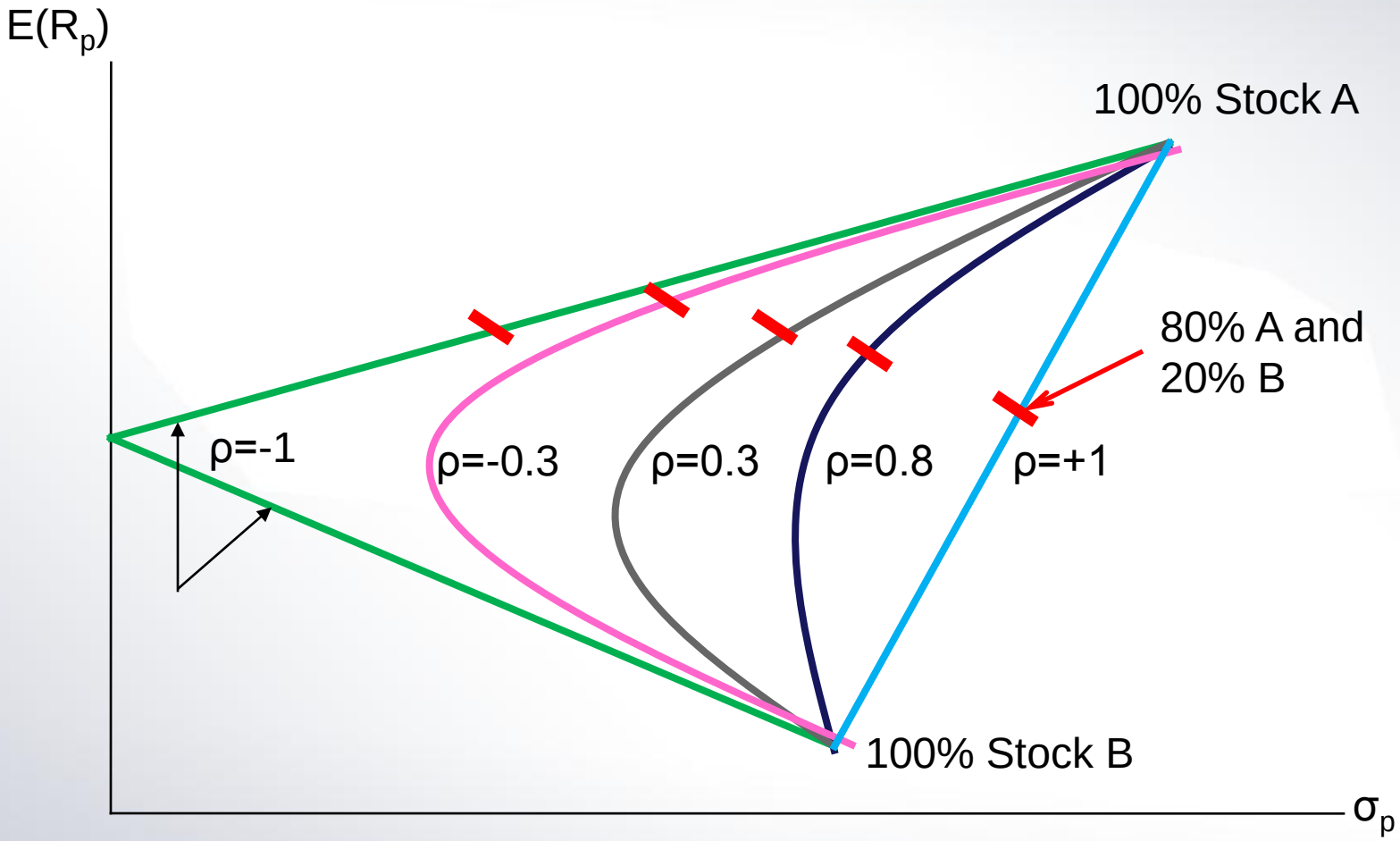
The Portfolio Possibilities Curve



Minimum Variance Portfolio



Correlation and Portfolio Diversification



真题回顾

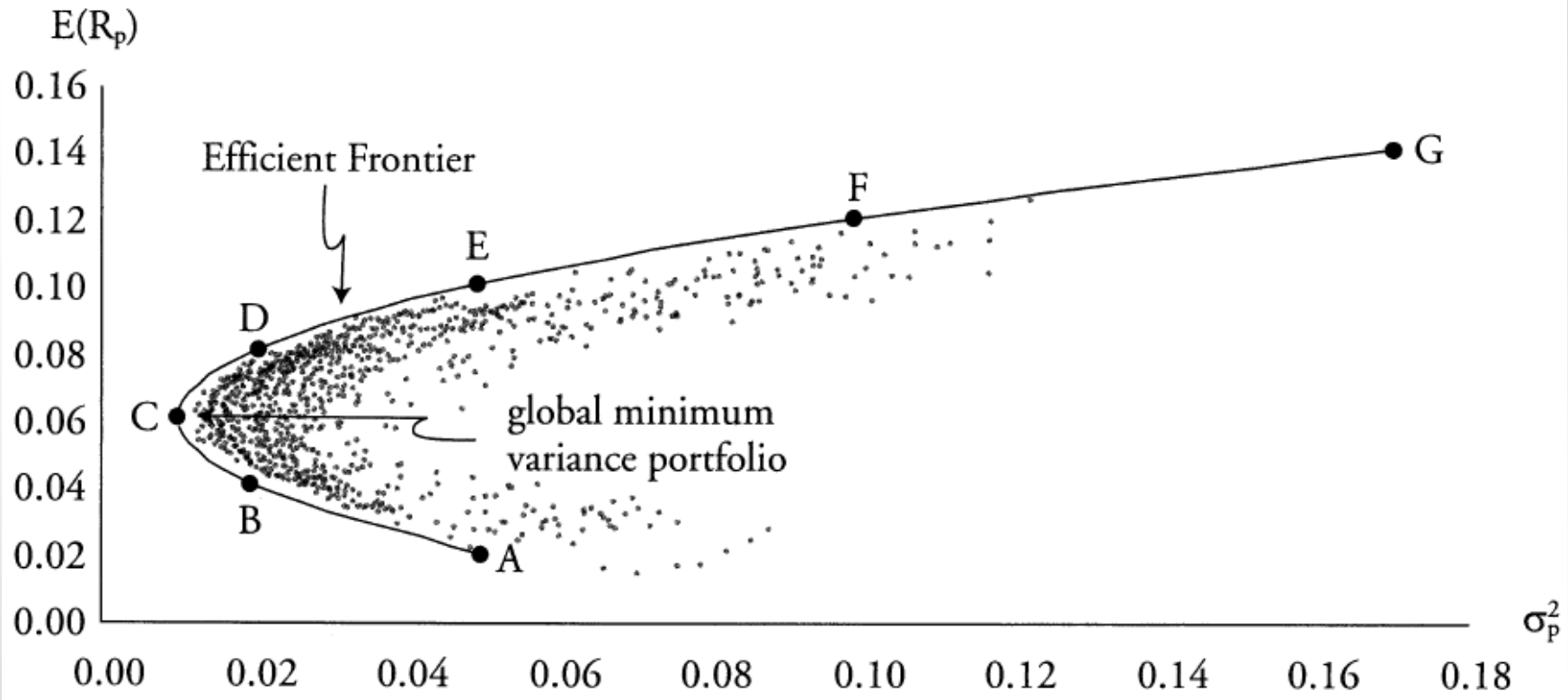
45. A risk manager is analyzing the characteristics of a portfolio created by combining two stocks with standard deviations of returns of 14% and 19%, and a correlation coefficient of -1 between their returns. Assuming no borrowing and no short selling is allowed, which of the following statements about potential portfolios created from only these two stocks is correct?

A. It is possible to create a portfolio that has a standard deviation of returns greater than 19%.
B. It is possible to create a portfolio that has a standard deviation of returns of 0%.
C. All possible portfolios will lie on the efficient frontier in standard deviation/return space.
D. All possible portfolios will lie on a single straight line in standard deviation/return space.

Answer: B

The Efficient Frontier

Figure 8: Efficient Frontier



The piece of the portfolio possibilities curve that lies above the minimum variance portfolio (from point C through point G) is **concave (is efficient)**.
The piece of the portfolio possibilities curve that lies below the minimum variance portfolio (from point A through point C) is **convex (is not efficient)**.

Short Sales and the Efficient Frontier

Figure 9: Portfolio Returns for Various Weights of Two Assets (w/ Short Sales)

$w_{\text{Caffeine Plus}}$	100%	80%	60%	40%	20%	0%	–20%	–40%	– 60%	–80%	– 100%
$w_{\text{Sparklin'}}$	0%	20%	40%	60%	80%	100%	120%	140%	160%	180%	200%
$\hat{R}_p$	11.00%	13.80%	16.60%	19.40%	22.20%	25.00%	27.80%	30.60%	33.40%	36.20%	39.00%
σ_p	15.00%	13.74%	13.72%	14.94%	17.10%	20.00%	23.28%	26.82%	30.53%	34.36%	38.28%

- When allowing for short sales, the efficient frontier expands up and to the right.
- By shorting, it is possible to create higher return and higher volatility portfolio combinations that would not be possible otherwise.
- Theoretically, with no limitations on shorting, it would be possible to construct a portfolio with infinite return.

Examples

- When short sales are possible (i.e., there are no short sale restrictions), the efficient frontier is:

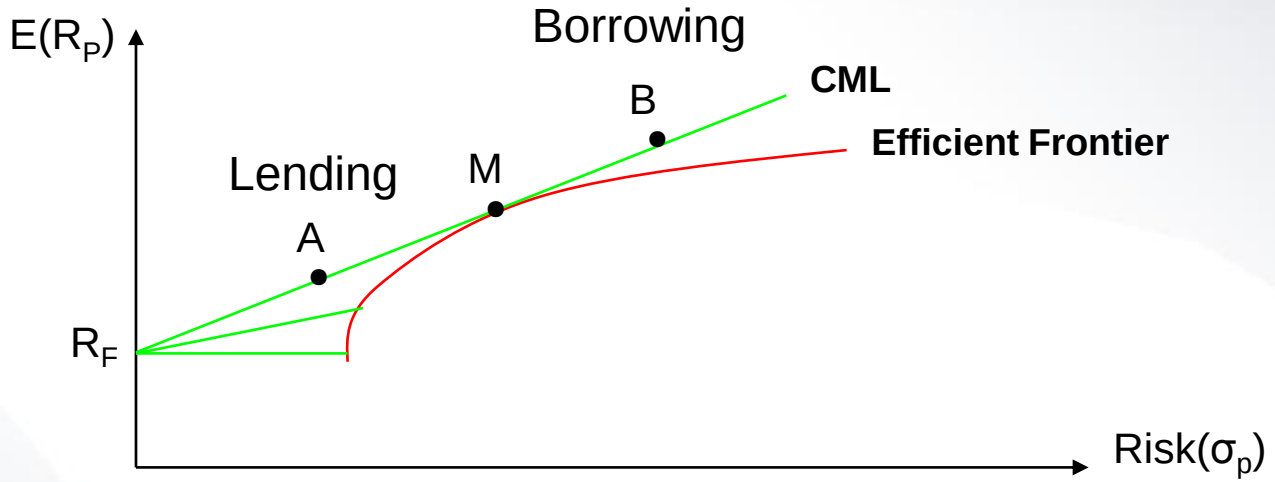
A. a straight line between the risk-free asset and the market portfolio.

B. two line segments, which indicate a negative relationship between short and long positions.

C. expanded to include portfolios with higher return and lower volatility.

D. expanded to include portfolios with higher return and higher volatility.

➢ Capital Market Line (CML)



- $E(R_P) = w_F R_F + w_M E(R_M)$
- $\sigma_p = w_M \sigma_M$
  - ✓ Capital Market Line: $E(R_P) = R_F + \left(\frac{E(R_M) - R_F}{\sigma_M} \right) \times \sigma_p$

➢ *Notes*:

- After the addition of risk-free asset, the allocation decision is between the riskless asset and the market portfolio. The CML is efficient frontier.

Examples

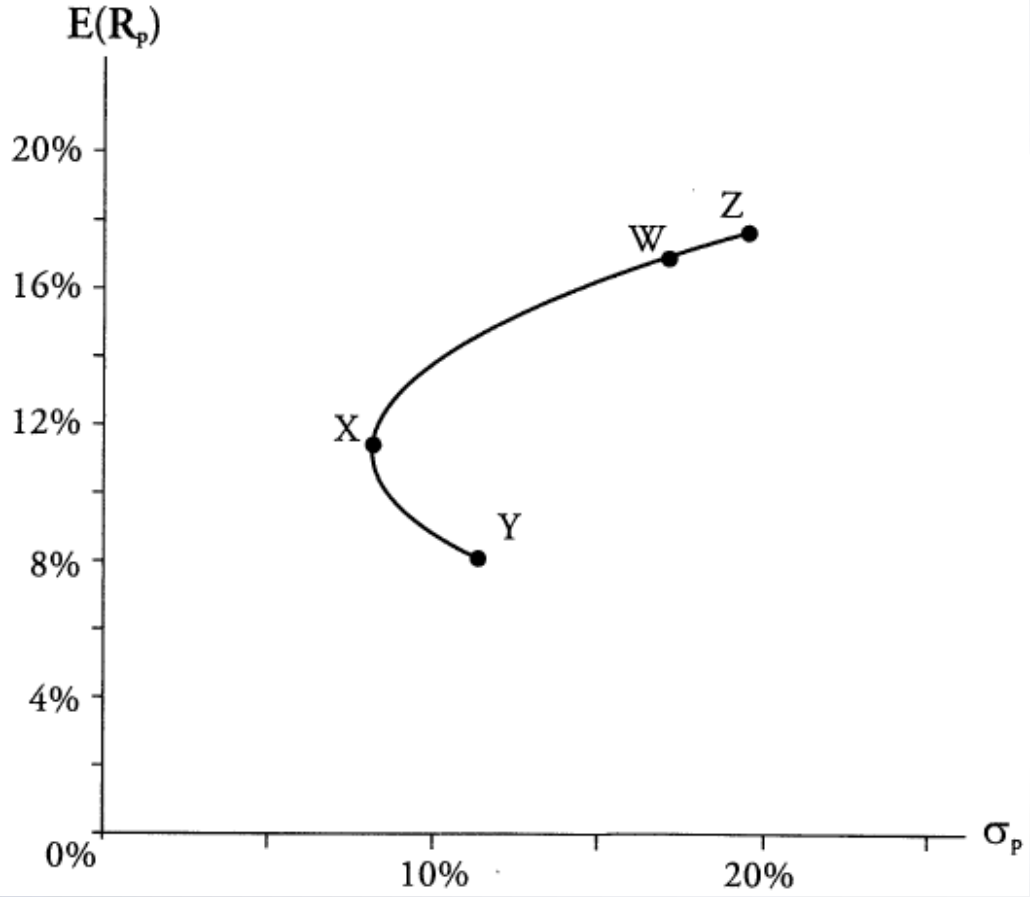
1. Assume the following information for stocks A and B.
   - Expected return on Stock A = 18%.
   - Expected return on Stock B = 23%.
   - Correlation between returns of Stock A and Stock B = 0.10.
   - Standard deviation of returns on Stock A = 40%.
   - Standard deviation of returns on Stock B = 50%.

   The expected return and standard deviation of an equally weighted portfolio of stocks A and B are closest to:

	Expected return (%)	Standard deviation (%)
A.	20.5	33.54
B.	20.5	11.22
C.	33.5	11.22
D.	33.5	33.54

Examples

- Use the following data to answer Questions 2 and 3.
  - Assume the expected return on stocks is 18% (represented by *Z* in the figure), and the expected return on bonds is 8% (represented by point *Y* on the graph).



Examples

2. The graph shows the portfolio possibilities curve for stocks and bonds. The point on the graph that most likely represents a 90% allocation in stocks and a 10% allocation in bonds is Portfolio:

   A. W.
   B. X.
   C. Y.
   D. Z.

3. The efficient frontier consists of the portfolios between and including:

   A. X and W
   B. Y and Z.
   C. X and Z.
   D. Y and X.

Examples

4. Which of the following best describes the shape of the portfolio possibilities curve?

A. The curve is strictly convex.

B. The curve is strictly concave.

C. The curve is concave above the minimum variance portfolio and convex below the minimum variance portfolio.

D. The curve is convex above the minimum variance portfolio and concave below the minimum variance portfolio.

真题回顾

33. An investor holds a portfolio of stocks A and B. The current value, estimated annual expected return, and estimated annual standard deviation of returns are summarized in the table below:

	Stock A	Stock B
Current Value (USD)	40,000	60,000
Expected Return	8%	9%
Standard Deviation	16%	20%

If the correlation coefficient of the returns on stocks A and B is 0.3, then the expected value of the portfolio at the end of this year, within two standard deviations, will be between:

A. USD 69,600 and USD 130,400.
B. USD 71,800 and USD 145,400.
C. USD 78,200 and USD 139,000.
D. USD 81,400 and USD 135,800.

真题解答

答案：C

$$E(R_P) = 0.4 \times 8\% + 0.6 \times 9\% = 8.6\%$$

$$\sigma_P = \sqrt{0.4^2 \times 16\%^2 + 0.6^2 \times 20\%^2 + 2 \times 0.4 \times 0.6 \times 0.3 \times 16\% \times 20\%} = 15.2\%$$

两个标准差的收益变动范围：

$$8.6\% - 2 \times 15.2\%, \quad 8.6\% + 2 \times 15.2\%$$

$$\rightarrow (-21.8\%, \quad 39\%)$$

预期的价值范围：

$$100,000 \times (1 - 21.8\%, \quad 1 + 39\%)$$

$$\rightarrow (78,200, \quad 139,000)$$

恭祝大家

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