
Probabilities

概率

Random Variable

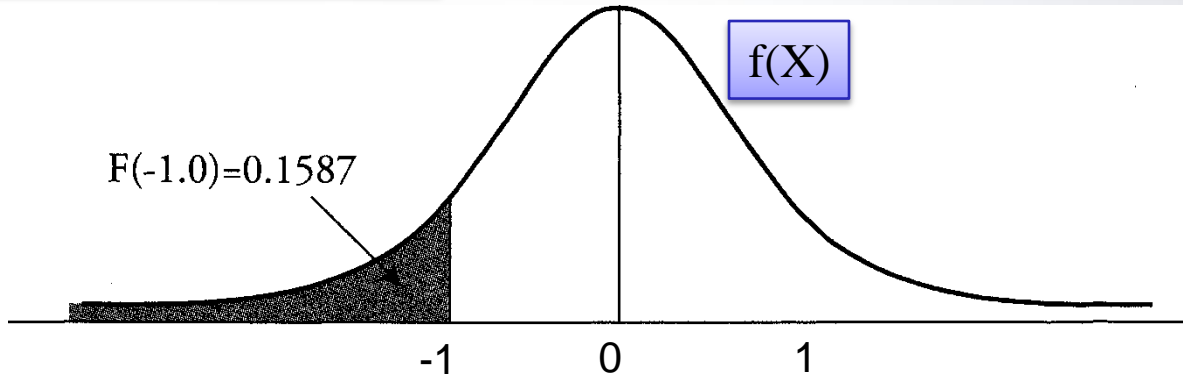
- **Random variable** (随机变量): is an uncertain quantity/number.
- **Outcome** (结果): is an observed value of a random variable.
- **Event** (事件): is a single outcome or a set of outcomes.
 - **Mutually exclusive events** (互斥事件): are events that cannot both happen at the same time.
 - **Exhaustive events** (遍历事件): are those that include all possible outcomes.
- **Probability distribution**: describes the probabilities of all the possible outcomes for a random variable.
- **Discrete random variable** and **continuous random variable**

Discrete Random Variable and Probability Function

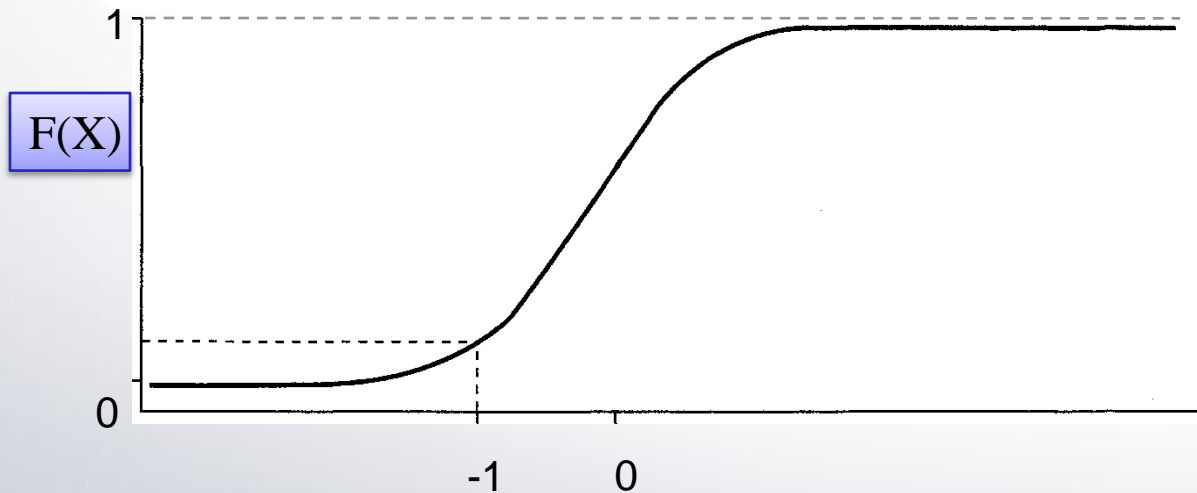
- A **probability function**, denoted $p(x)$, specifies the probability that a random variable is equal to a specific value. More formally, $p(x)$ is the probability that random variable X takes on the value x , or $p(x) = P(X = x)$.
- The two key properties of a probability function are:
 - $0 \leq p(x) \leq 1$.
 - $\sum p(x) = 1$, the sum of the probabilities for all possible outcomes, x , for a random variable, X , equals 1.

Continuous Random Variable and Distribution Function

(a) Probability density function



(b) Cumulative distribution function

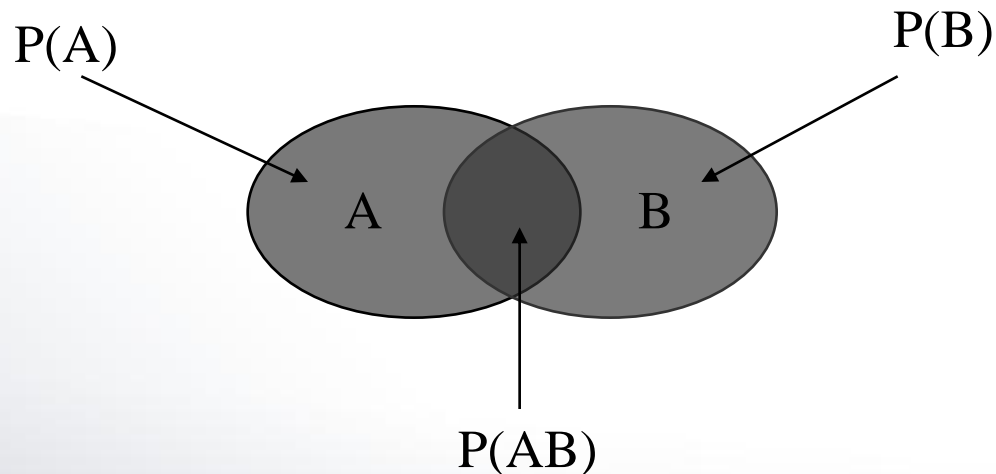


Unconditional Probability and Conditional Probability

➤ Conditional probability 条件概率: $P(A|B) = \frac{P(AB)}{P(B)}$

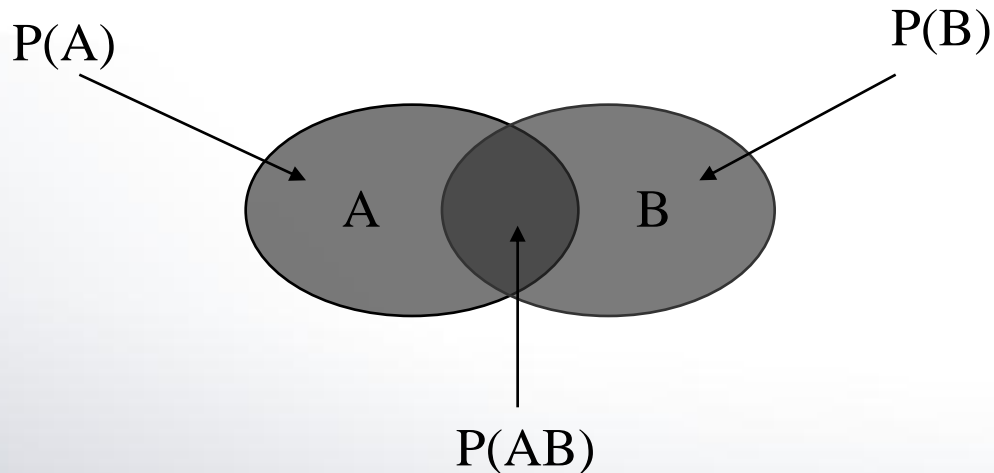
➤ Joint probability 联合概率: $P(AB)$

➤ Multiplication rule: $P(AB) = P(A|B) \times P(B)$



Independent and Mutually Exclusive Events

- The occurrence of A has no influence of on the occurrence of B
 - $P(A|B)=P(A)$ or $P(B|A)=P(B)$
 - $P(AB)=P(A)\times P(B)$
 - $P(A \text{ or } B)=P(A)+P(B)-P(A)\times P(B)$
- Addition rule: $P(A \text{ or } B)=P(A)+P(B)-P(AB)$
 - If A and B are mutually exclusive events, then: $P(A \text{ or } B)=P(A)+P(B)$



真题回顾

- Bond A and Bond B have the same rating and the same probability of default. It is also estimated that:
- The probability that both Bond A and Bond B will default during the next year is 5%; and
 - If Bond A defaults next year, there is a 50% probability that Bond B will also default.

What is the probability that neither Bond A nor Bond B will default over the next year?

- A. 75%
- B. 80%
- C. 85%
- D. 95%

$$P(AB) = 5\%; \quad P(B|A) = \frac{P(AB)}{P(A)} = \frac{5\%}{P(A)} = 50\% \rightarrow P(A) = 10\%$$

$$\begin{aligned} P(\overline{A \cup B}) &= 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(AB)] \\ &= 1 - (10\% + 10\% - 5\%) = 85\% \end{aligned}$$

Probability Matrix

➤ Joint and Unconditional Probabilities

		<i>Interest Rates</i>		
		Increase	No Increase	
<i>Economy</i>	Good	14%	6%	20%
	Normal	20%	30%	50%
	Poor	6%	24%	30%
		40%	60%	100%

EXAMPLE 2.2: FRM EXAM 2007—QUESTION 93

The joint probability distribution of random variables X and Y is given by $f(x, y) = k \times x \times y$ for $x = 1, 2, 3$, $y = 1, 2, 3$, and k is a positive constant. What is the probability that $X + Y$ will exceed 5?

- a. $1/9$
- b. $1/4$
- c. $1/36$
- d. Cannot be determined

Answer: b. The function $x \times y$ is described in the following table. The sum of the entries is 36. The scaling factor k must be such that the total probability is one. Therefore, we have $k = 1/36$. The table shows one instance where $x + y > 5$, which is $x = 3$, $y = 3$. The probability is $p = 9/36 = 1/4$.

$x \times y$	$x = 1$	2	3
$y = 1$	1	2	3
2	2	4	6
3	3	6	9

Bayes' Formula

➤ $P(AB) = P(A|B) \times P(B) = P(B|A) \times P(A)$

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

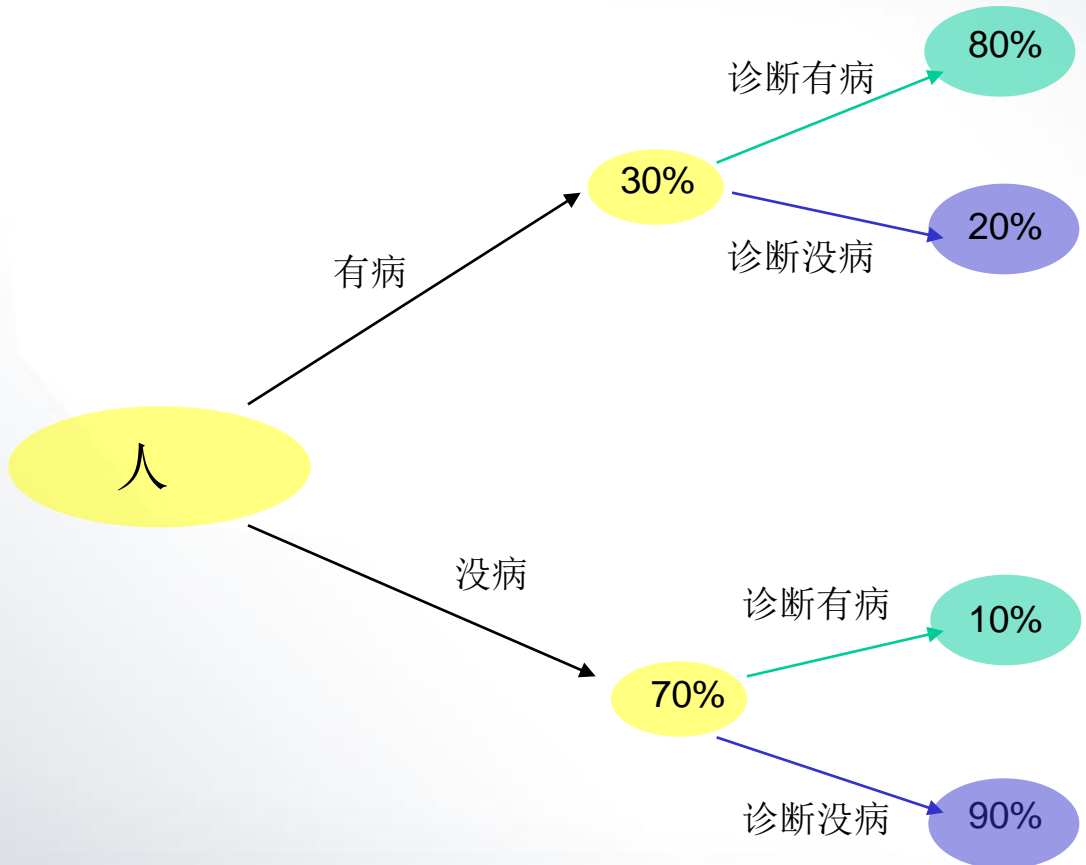
➤ Example:

快速诊断仪: 有病 0.3 没病 0.7

	机器说有病	机器说没病
如果人真有病	0.8	0.2
如果人真没病	0.1	0.9

现一个人诊断为有病，问其真有病的概率？

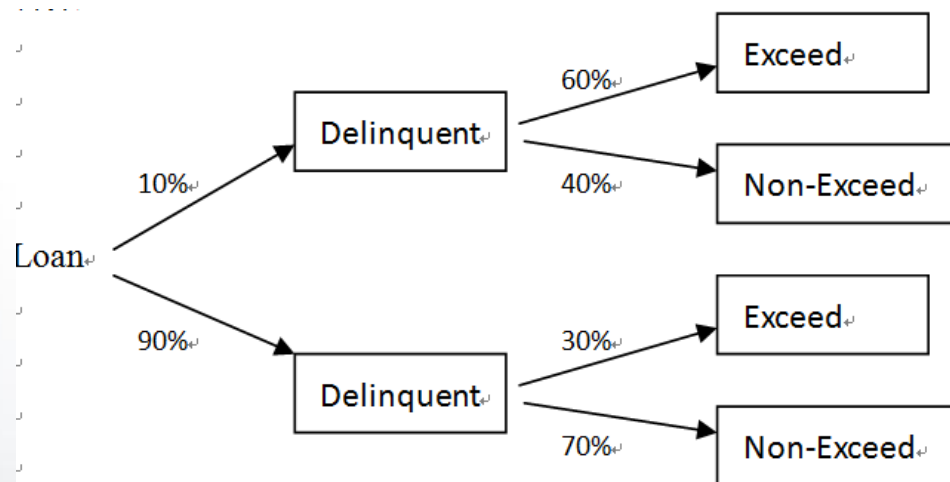
Bayes' Formula



真题回顾

- You are examining a pool of senior secured loans and observe that 10% of the loans are delinquent (拖欠) in their interest payments. The outstanding balance on 60% of the delinquent loans exceeds the value of the collateral pledged to secure them and the outstanding balance on 30% of the non-delinquent loans exceeds the value of the collateral pledged to secure them. If you randomly select a loan from the pool and observe that its collateral value is less than the outstanding balance, what is the probability that the loan is delinquent?

- A. 6%
- B. 9%
- C. 18%
- D. 54%



$$P(\text{Delinquent} | \text{Exceed}) = \frac{10\% \times 60\%}{10\% \times 60\% + 90\% \times 30\%} = 18\%$$

恭祝大家

FRM学习愉快！

顺利通过考试！