Non-Parametric Approaches

非参数方法

Basic Historical Simulation VaR and ES

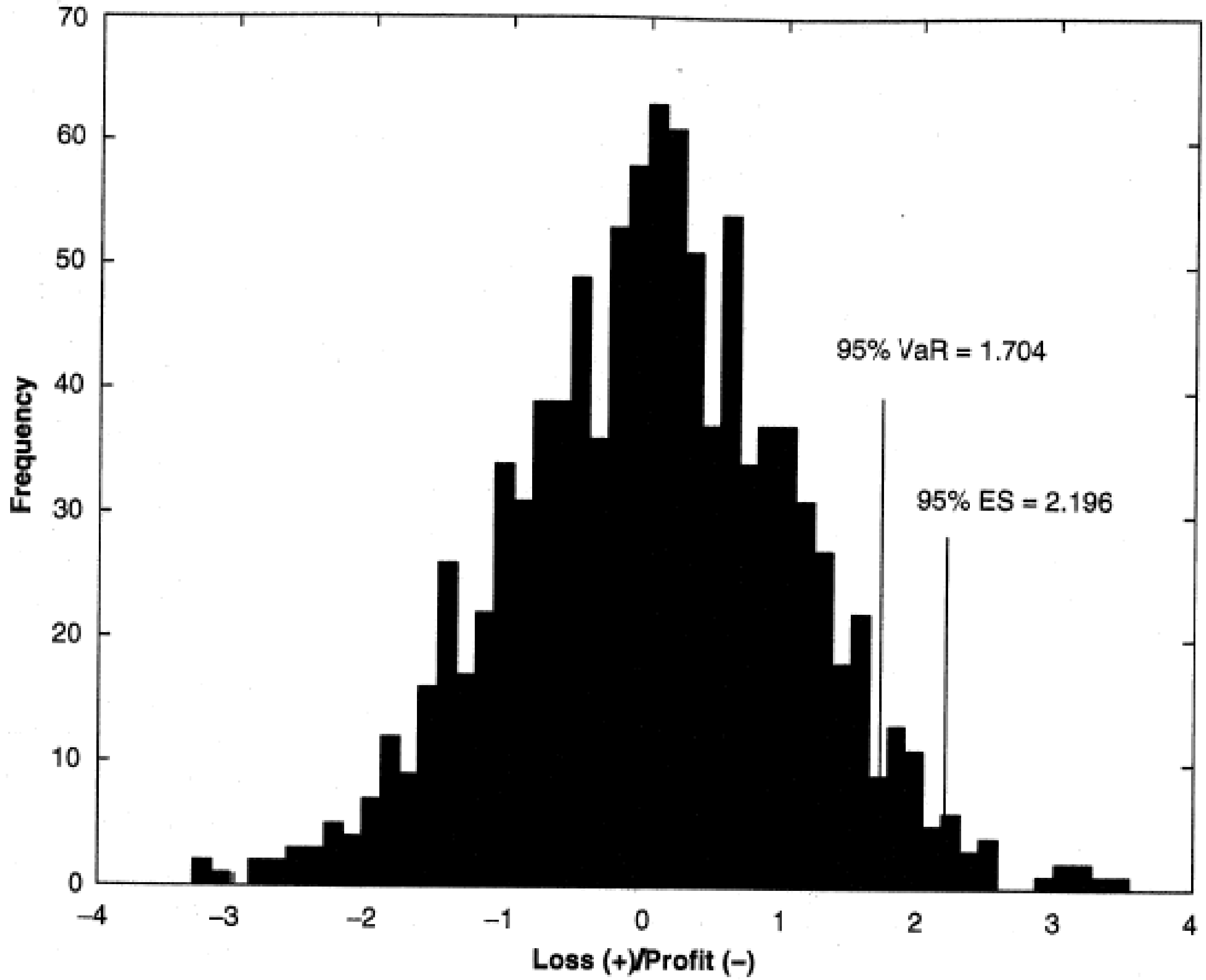


FIGURE 2-1 Basic historical simulation VaR and ES.

Bootstrap Historical Simulation Approach

- The bootstrap historical simulation is a simple and intuitive estimation procedure. In essence, the bootstrap technique draws a sample from the original data set, records the VaR from that particular sample and "returns" the data. This procedure is repeated over and over and records multiple sample VaRs. Since the data is always "returned" to the data set, this procedure is akin to sampling with replacement. The best VaR estimate from the full data set is the average of all sample VaRs.

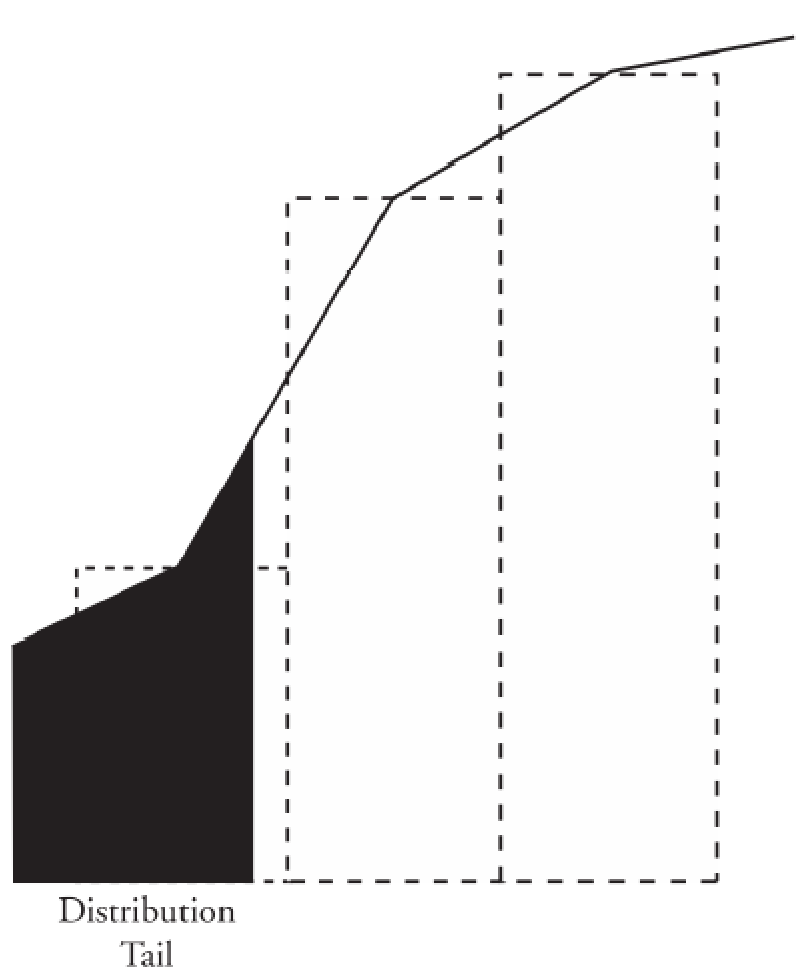
- This same procedure can be performed to estimate the expected shortfall (ES). Each drawn sample will calculate its own ES by slicing the tail region into n slices and averaging the VaRs at each of then n- 1 quantiles. This is exactly the same procedure described in the previous topic. Similarly, the best estimate of the expected shortfall for the original data set is the average of all of the sample expected shortfalls.

- Empirical analysis demonstrates that the bootstrapping technique consistently provides more precise estimates of coherent risk measures than historical simulation on raw data alone.

Historical Simulation Using Non-Parametric Density Estimation

- The clear advantage of the traditional historical simulation approach is its simplicity. One obvious drawback, however, is that the discreteness (离散性) of the data does not allow for estimation of VaRs between data points. If there were 100 historical observations, then it is straightforward to estimate VaR at the 95% or the 96% confidence levels, and so on. However, this method is unable to incorporate a confidence level of 95.5%, for example. More generally, with n observations, the historical simulation method only allows for n different confidence levels.

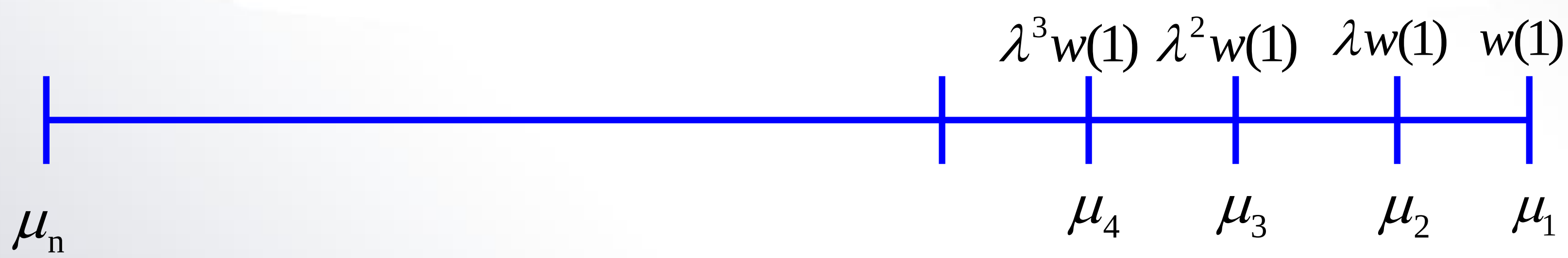
Figure 1: Surrogate Density Function



Weighted Historical Simulation Approaches

1. Age-weighted Historical Simulation

The obvious adjustment to the equal-weighted assumption used in historical simulation is to weight recent observations more and distant observations less. One method proposed by Boudoukh, Richardson, and Whitelaw is as follows.[1] Assume w(1) is the probability weight for the observation that is one day old. Then w(2) can be defined as λw(1), w(3) can be defined as λ^2w(1), and so on. The decay parameter, λ, can take on values $0 \le \lambda \le 1$ where values close to 1 indicate slow decay. Since all of the weights must sum to 1, we conclude that $w(1) = (1 - \lambda) / (1 - \lambda^n)$. More generally, the weight for an observation that is *i* days old is equal to:



$$w_{(1)} + \lambda w_{(1)} + \ldots + \lambda^{n-1} w_{(1)} = 1 \rightarrow w_{(i)} = \frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^n}$$

Weighted Historical Simulation Approaches

2. Volatility-weighted Historical Simulation

通过当前的波动和以前的波动来调整以前的收益 （主要是调收益）

$$r_{t,i}^{*}=\left(\frac{\sigma_{T,i}}{\sigma_{t,i}}\right)\times r_{r,i}$$

where: $r_{r,i}$ = actual return for asset i on day t

$\sigma_{t,i}$ = volatility forecast for asset i on day t (made at the end of day t-1)

$\sigma_{T,i}$ = current forecast of volatility for asset i

There are several advantages of the volatility-weighted method. First, it explicitly incorporates volatility into the estimation procedure in contrast to other historical methods. Second, the near-term VaR estimates are likely to be more sensible in light of current market conditions. Third, the volatility-adjusted returns allow for VaR estimates that are higher than estimates with the historical data set.

Weighted Historical Simulation Approaches

3. Correlation-weighted Historical Simulation

- As the name suggests, this methodology incorporates updated correlations between asset pairs. This procedure is more complicated than the volatility-weighted approach, but it follows the same basic principles. Intuitively, the historical correlation (or equivalently variance-covariance) matrix needs to be adjusted to the new information environment. *This is accomplished, loosely speaking, by "multiplying" the historic returns by the revised correlation matrix to yield updated correlation-adjusted returns*.

4. Filtered Historical Simulation

- The *filtered historical simulation* is the most *comprehensive*, and hence most *complicated*, of the non-parametric estimators.
- The process combines the *historical simulation model* with *conditional volatility models* (like GARCH or asymmetric GARCH). Thus, the method contains both the attractions of the traditional historical simulation approach with the sophistication of models that incorporate changing volatility.
- In simplified terms, the model is flexible enough to capture conditional volatility and volatility clustering as well as a surprise factor that could have an asymmetric effect on volatility.

Advantages and Disadvantages of Non-Parametric Methods

- **Advantages of non-parametric methods include the following:**
  - Intuitive and often computationally simple (even on a spreadsheet).
  - Not hindered by parametric violations of skewness, fat-tails, et cetera.
  - Avoids complex variance-covariance matrices and dimension problems.
  - Data is often readily available and does not require adjustments (e.g., financial statements adjustments).
  - Can accommodate more complex analysis (e.g., by incorporating age-weighting with volatility-weighting).

Advantages and Disadvantages of Non-Parametric Methods

- **Disadvantages of non-parametric methods include the following:**
  - Analysis depends critically on historical data.
  - Volatile data periods lead to VaR and ES estimates that are too high.
  - Quiet data periods lead to VaR and ES estimates that are too low.
  - Difficult to detect structural shifts/regime changes in the data.
  - Cannot accommodate plausible (貌似真实的) large impact events if they did not occur within the sample period.
  - Difficult to estimate losses significantly larger than the maximum loss within the data set (historical simulation cannot; volatility-weighting can, to some degree).
  - Need sufficient data, which may not be possible for new instruments or markets.

Examples

1. Johanna Roberto has collected a data set of 1,000 daily observations on equity returns. She is concerned about the appropriateness of using parametric techniques as the data appears skewed. Ultimately, she decides to use historical simulation and bootstrapping to estimate the 5% VaR. Which of the following steps is most likely to be part of the estimation procedure?
   - A. Filter the data to remove the obvious outliers.
   - B. Repeated sampling with replacement.
   - C. Identify the tail region from reordering the original data.
   - D. Apply a weighting procedure to reduce the impact of older data.

B Bootstrapping from historical simulation involves repeated sampling with replacement. The 5% VaR is recorded from each sample draw. The average of the VaRs from all the draws is the VaR estimate. The bootstrapping procedure does not involve filtering the data or weighting observations. Note that the VaR from the original data set is not used in the analysis.

Examples

2. All of the following approaches improve the traditional historical simulation approach for estimating VaR except the:

   A. volatility-weighted historical simulation.

   B. age-weighted historical simulation.

   C. market-weighted historical simulation.

   D. correlation-weighted historical simulation.

C Market-weighted historical simulation is not discussed in this topic. Age-weighted historical simulation weights observations higher when they appear closer to the event date. Volatility-weighted historical simulation adjusts for changing volatility levels in the data. Correlation-weighted historical simulation incorporates anticipated changes in correlation between assets in the portfolio.

Examples

3. Which of the following statements about age-weighting is most accurate?

A. The age-weighting procedure incorporates estimates from GARCH models.

B. If the decay factor in the model is close to 1, there is persistence within the data set.

C. When using this approach, the weight assigned on day i is equal to:

$$w(i) = \lambda^{i-1} \times (1-\lambda) / \left(1-\lambda^{i}\right)$$

D. The number of observations should at least exceed 250.

B If the intensity parameter (i.e., decay factor) is close to 1, there will be persistence (i.e., slow decay) in the estimate. The expression for the weight on day i has i in the exponent when it should be n. While a large sample size is generally preferred, some of the data may no longer be representative in a large sample.

Examples

4. Which of the following statements about volatility-weighting is true?
   A. Historic returns are adjusted, and the VaR calculation is more complicated.
   B. Historic returns are adjusted, and the VaR calculation procedure is the same.
   C. Current period returns are adjusted, and the VaR calculation is more complicated.
   D. Current period returns are adjusted, and the VaR calculation is the same.

B The volatility-weighting method adjusts historic returns for current volatility. Specifically, return at time *t* is multiplied by (current volatility estimate / volatility estimate at time *t*). However, the actual procedure for calculating VaR using a historical simulation method is unchanged; it is only the inputted data that changes.

Examples

5. All of the following items are generally considered advantages of non-parametric estimation methods except:
   - A. Ability to accommodate skewed data.
   - B. availability of data.
   - C. use of historical data.
   - D. little or no reliance on covariance matrices.

C The use of historical data in non-parametric analysis is a disadvantage, not an advantage. If the estimation period was quiet (volatile) then the estimated risk measures may understate (overstate) the current risk level. Generally, the largest VaR cannot exceed the largest loss in the historical period. On the other hand, the remaining choices are all considered advantages of non-parametric methods. For instance, the non-parametric nature of the analysis can accommodate skewed data, data points are readily available, and there is no requirement for estimates of covariance matrices.

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