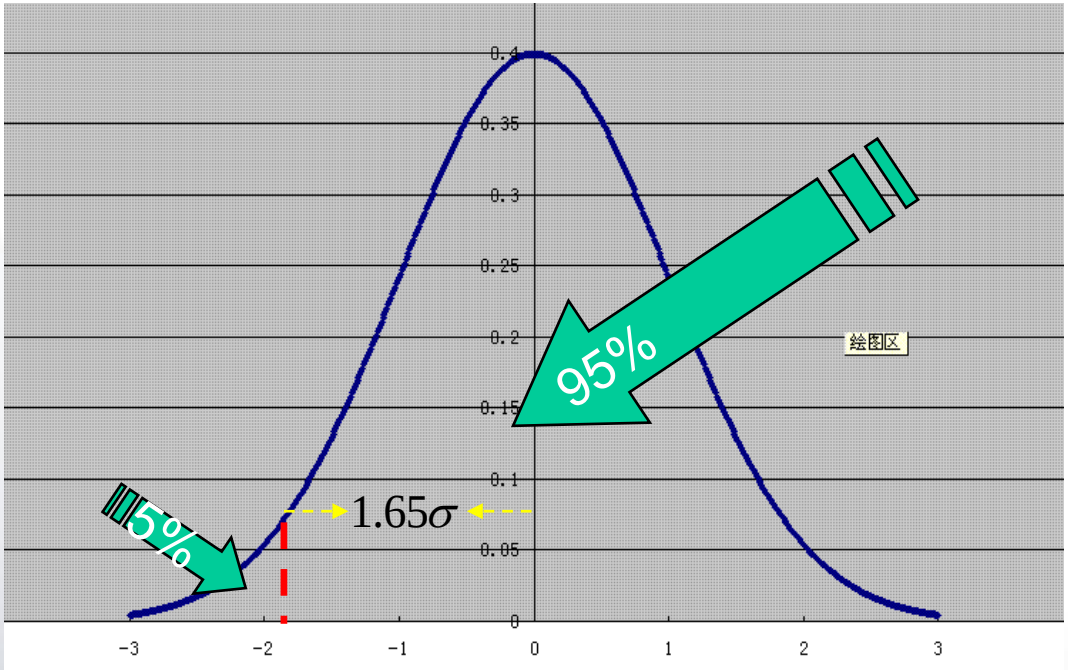
VaR Methods

VaR 方法

Defining VaR

➢ The Definition:

- Value at Risk (VaR) is the **maximum loss** over a **target horizon** and for a **given confidence level**.
- VAR是在一定持有期内，在一定的置信水平下的最大损失。



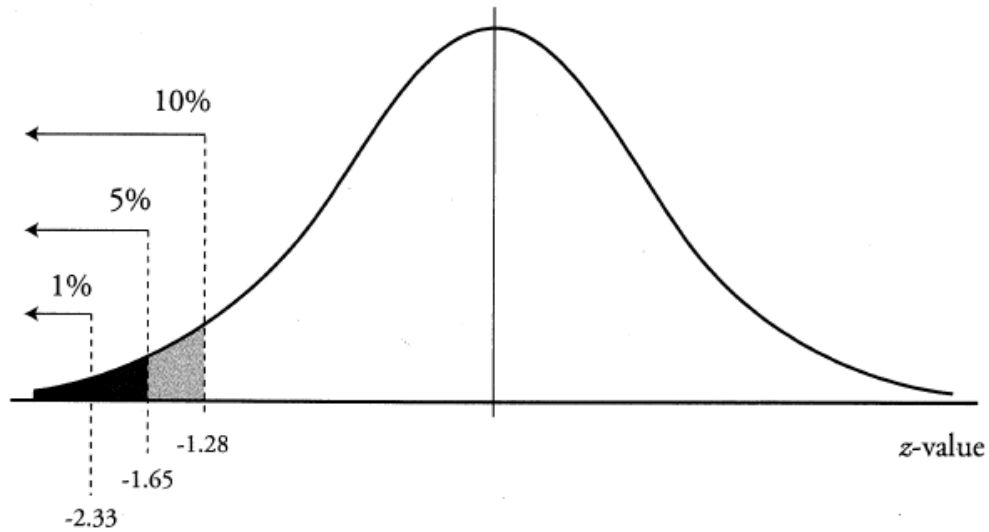
Defining VaR

- A brief example will help solidify the VaR concept.
  - Assume a risk manager calculates the daily 5% VaR as $10,000. The VaR(5%) of $10,000 indicates that there is a 5% chance that on any given day, the portfolio will experience a loss of $10,000 or more. We could also say that there is a 95% chance that on any given day the portfolio will experience either a loss less than $10,000 or a gain.
  - If we further assume that the $10,000 loss represents 8% of the portfolio value, then on any given day there is a 5% chance that the portfolio will experience a loss of 8% or greater, but there is a 95% chance that the loss will be less than 8% or a percentage gain greater than zero.

Calculating VaR

- Delta-Normal VaR

Figure 1: Standard Normal Distribution and Cumulative Probabilities



Percent VaR: $$\mathrm{VaR}(X\%) = z_{X\%} \times \sigma$$

Dollar VaR: $$\mathrm{VaR}(X\%) = z_{X\%} \times \sigma \times \text{asset value}$$

Calculating VaR

Example: Calculating percentage and dollar VaR

A risk management officer at a bank is interested in calculating the VaR of an asset that he is considering adding to the bank's portfolio. If the asset has a daily standard deviation of returns equal to 1.4% and the asset has a current value of $5.3 million, **calculate** the VaR (5%) on both a percentage and dollar basis.

Answer:

The appropriate critical *z*-value for a VaR (5%) is –1.65. Using this critical value and the asset's standard deviation of returns, the VaR (5%) on a percentage basis is calculated as follows:

$$\text{VaR (5\%)} = z_{5\%}\sigma = -1.65(0.014) = -0.0231 = -2.31\%$$

The VaR(5%) on a dollar basis is calculated as follows:

$$\begin{aligned}\text{VaR (5\%)}_{\text{dollar basis}} &= \text{VaR (5\%)}_{\text{decimal basis}} \times \text{asset value} = -0.0231 \times \$5{,}300{,}000 \\ &= -\$122{,}430\end{aligned}$$

Thus, there is a 5% probability that, on any given day, the loss in value on this particular asset will equal or exceed 2.31%, or $122,430.

Calculating VaR

- If an expected return other than zero is given, VaR becomes the expected return minus the quantity of the critical value multiplied by the standard deviation.

$$\boxed{\mathrm{VaR} = \left[\mathrm{E(R)} - z\sigma\right]}$$

Example: Calculating VaR given an expected return

For a $100,000,000 portfolio, the expected 1-week portfolio return and standard deviation are 0.00188 and 0.0125, respectively. **Calculate** the 1-week VaR at 5% significance.

Answer:

$$\begin{aligned}\mathrm{VaR} &= [\mathrm{E(R)} - z\sigma] \times \text{portfolio value}\\ &= [0.00188 - 1.65(0.0125)] \times \$100{,}000{,}000\\ &= -0.018745 \times \$100{,}000{,}000\\ &= -\$1{,}874{,}500\end{aligned}$$

The manager can be 95% confident that the maximum 1-week loss will not exceed $1,874,500.

VaR Conversions

- **Square Root Rule**

$$\mathrm{VaR}(X\%)_{\text{J-days}} = \mathrm{VaR}(X\%)_{\text{1-day}} \times \sqrt{J}$$

- **Converting VaR to different confidence levels**

$$\mathrm{VaR}(1\%) = \mathrm{VaR}(5\%) \times \frac{z_{1\%}}{z_{5\%}} = \mathrm{VaR}(5\%) \times \frac{2.33}{1.65}$$

真题回顾

1. Assume that the P&L distribution of a liquid asset is i.i.d. normally distributed. The position has a one-day VAR at the 95% confidence level of $100,000. Estimate the 10-day VAR of the same position at the 99% confidence level.

   A. $1,000,000

   B. $450,000

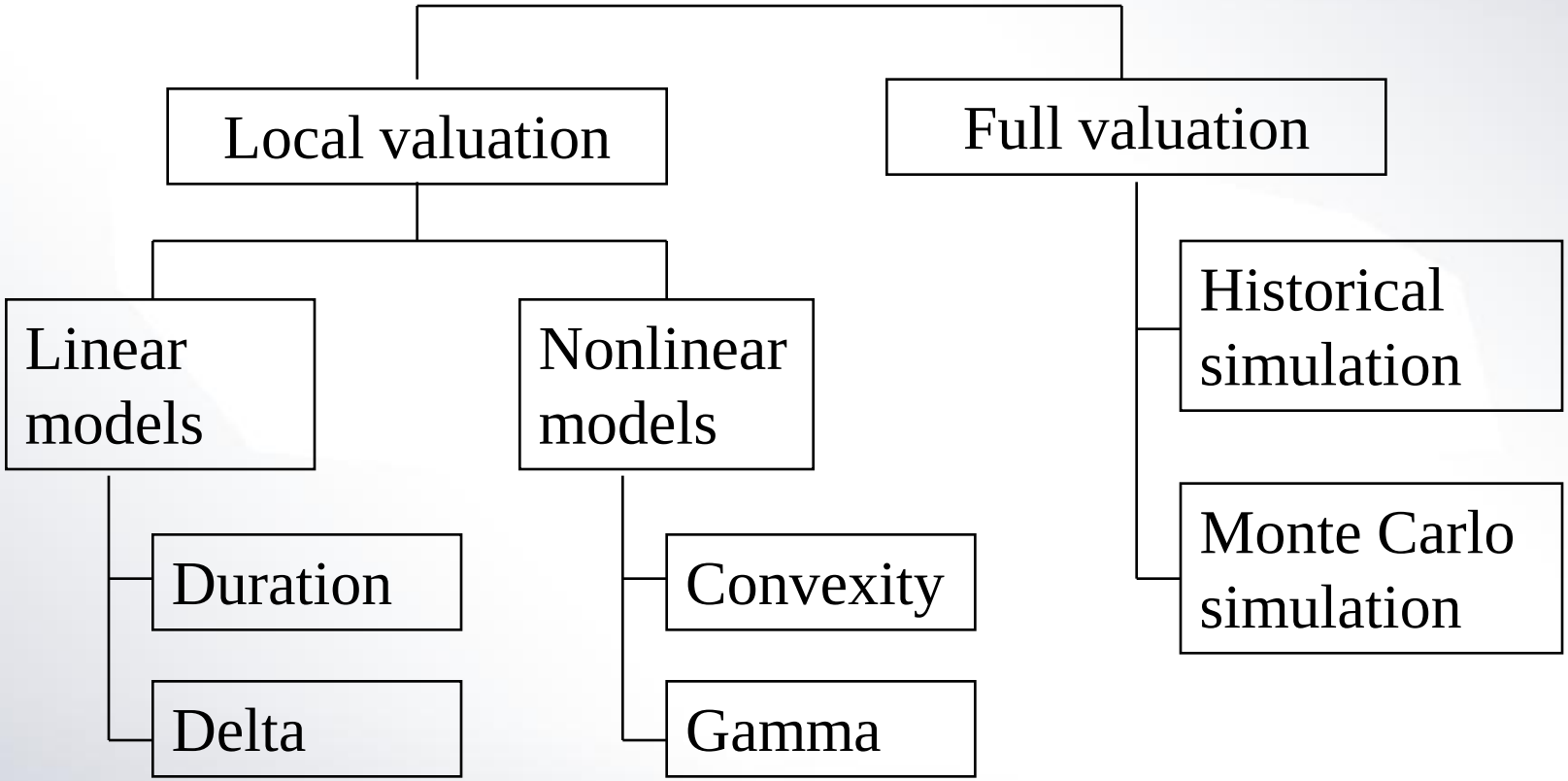
   C. $320,000

   D. $220,000

Example 12.6: FRM Exam 2008—Question 2-2

b. We need to scale the VAR to a 99% level using $\$100,000 \times 2.326/1.645 = \$141,398$. Multiplying by $\sqrt{10}$ then gives $447,140.

VaR Methods


Local valuation
Full valuation
Linear models
Nonlinear models
Historical simulation
Monte Carlo simulation
Duration
Delta
Convexity
Gamma

The VaR Methods

- **Local Valuation Linear models (Delta Normal)**

$$VAR(dP) = |-D^{*}P| \times VAR(dy)$$

$$VAR(df) = |\Delta| \times VAR(dS)$$

- **Local Valuation Nonlinear models (Delta-Gamma)**

$$VAR(dP) = |-D^{*}P| \times VAR(dy) - (1/2)(C \times P) \times VAR(dy)^{2}$$

$$VAR(df) = |\Delta| \times VAR(dS) - (1/2)\,\Gamma \times VAR(dS)^{2}$$

19. A portfolio manager bought 1,000 call options on a non-dividend-paying stock with a strike price of USD 100 for USD 5 each. The current stock price is USD 104 with a daily stock return volatility of 2.89%, and the delta of the option is 0.7. Using the delta-normal approach to calculate VaR, what is the approximate 1-day 95% VaR of this option?

A. USD 238
B. USD 3,461
C. USD 4,944
D. USD 7,063

19. A portfolio manager bought 1,000 call options on a non-dividend-paying stock with a strike price of USD 100 for USD 5 each. The current stock price is USD 104 with a daily stock return volatility of 2.89%, and the delta of the option is 0.7. Using the delta-normal approach to calculate VaR, what is the approximate 1-day 95% VaR of this option?

A. USD 238
B. USD 3,461
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D. USD 7,063

$$\mathrm{VaR}_{\text{option}} = \mathrm{VaR}_{\text{stock}} \times \Delta$$

$$= 1.645 \times 2.89\% \times 104 \times 0.7 \times 1000$$

$$= 3460.9484 \approx 3461$$

The VaR Methods

➢ Historical Simulation Method

Example: Historical VaR

You have accumulated 100 daily returns for your $100,000,000 portfolio. After ranking the returns from highest to lowest, you identify the lowest six returns:

–0.0011, –0.0019, –0.0025, –0.0034, –0.0096, –0.0101

Calculate daily value at risk (VaR) at 5% significance using the historical method.

Answer:

The lowest five returns represent the 5% lower tail of the "distribution" of 100 historical returns. The fifth lowest return (–0.0019) is the 5% daily VaR. We would say there is a 5% chance of a daily loss exceeding 0.19%, or $190,000.

201405真题讲解

22. A risk manager is estimating the 1-day 95% VaR on a domestic equity portfolio using a 100-day lookback period. The mean return, estimated from the historical data, is 0.0% with a standard deviation of 2.0%. The six most extreme negative returns over the lookback period, along with the time they occurred, are:

order	Return	Numble of Days Ago
1	-10.0%	95
2	-6.3%	17
3	-4.7%	65
4	-4.0%	4
5	-3.8%	5
6	-3.6%	30

Over a period of 10 days after the risk manager computed the portfolio's VaR, four new extreme declines occurred: -25.0%, -4.1%, -7.8% and -9.5%. On the other six days, the portfolio experienced positive returns. The risk manager must now update the previous VaR estimate to account for these changes. Assuming the portfolio has a current value of USD100 million, What is the updated 1-day 95% VaR using the historical approach?

A. USD 3.28 million
B. USD 4.70 million
C. USD 10.0 million
D. USD 25.0 million

更新 10 天，取消前第 95 天，所以新的排序是：

-25%，-9.5%，-7.8%，-6.3%，-4.7%，-4.1%, ...

$VAR_{1\text{-day},95\%} = 4.7\% \times 100\text{million} = \text{USD } 4.70 \text{ million}$

恭祝大家

FRM学习愉快！

顺利通过考试！