Portfolio Risk: Analytical Methods

投资组合风险：分析方法

Portfolio VaR

- **Diversified Portfolio VAR**
  - $VAR_P = Z_c \times \sigma_P \times P$
- **Individual VAR**
  - $VAR_i = Z_c \times \sigma_i \times P_i = Z_c \times \sigma_i \times w_i \times P$
- **For a two-asset portfolio**
  - VAR for uncorrelated positions($\rho=0$):
    - ✓ $VAR_P = (VAR_1^2 + VAR_2^2)^{1/2}$
  - Undiversified VAR($\rho=1$)
    - ✓ $VAR_P = VAR_1 + VAR_2$

Example

Example: Computing portfolio VaR (part 1)

An analyst computes the VaR for the two positions in her portfolio. The VaRs: VaR_1 = \$2.4 million and VaR_2 = \$1.6 million. **Compute** VaR_P if the returns of the two assets are uncorrelated.

Answer:

For uncorrelated assets:

$$VaR_P = \sqrt{VaR_1^{\,2} + VaR_2^{\,2}} = \sqrt{\left(2.4^2 + 1.6^2\right)(\$millions)^2} = \sqrt{8.32(\$millions)^2}$$

$$VaR_P = \$2.8844 \text{ million}$$

Example: Computing portfolio VaR (part 2)

An analyst computes the VaR for the two positions in her portfolio. The VaRs: VaR_1 = \$2.4 million and VaR_2 = \$1.6 million. **Compute** VaR_P if the returns of the two assets are perfectly correlated.

Answer:

For perfectly correlated assets:

$$VaR_P = VaR_1 + VaR_2 = \$2.4 \text{ million} + \$1.6 \text{ million} = \$4 \text{ million}$$

Portfolio VaR

- Compute the standard deviation and VAR of an equally weighted portfolio, with equal standard deviation σ and correlation ρ:
- 2 assets: $$\sigma_p^2=\left(\frac{1}{2}\right)^2\sigma^2+\left(\frac{1}{2}\right)^2\sigma^2+2\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\rho\sigma^2=\frac{1}{2}\sigma^2+\frac{1}{2}\rho\sigma^2=\frac{1}{2}(1+\rho)\sigma^2$$
- 3 assets: $$\sigma_p^2=\left(\frac{1}{3}\right)^2\sigma^2+\left(\frac{1}{3}\right)^2\sigma^2+\left(\frac{1}{3}\right)^2\sigma^2+3\times2\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\rho\sigma^2=\frac{1}{3}\sigma^2+\frac{2}{3}\rho\sigma^2=\frac{1}{3}(1+2\rho)\sigma^2$$
- 4 assets: $$\sigma_p^2=\left(\frac{1}{4}\right)^2\sigma^2\times4+6\times2\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\rho\sigma^2=\frac{1}{4}\sigma^2+\frac{3}{4}\rho\sigma^2=\frac{1}{4}(1+3\rho)\sigma^2$$
- n assets: $$\sigma_p^2=\frac{1}{n}[1+(n-1)\rho]\sigma^2 \qquad \sigma_p=\sigma\sqrt{\frac{1}{N}+\left(1-\frac{1}{N}\right)\rho}$$
- when $N\to+\infty$ $$\sigma_p=\sigma\sqrt{\rho} \qquad VAR_P=z_c\cdot\sigma\sqrt{\rho}\cdot P$$

Example

Example: Computing portfolio VaR (part 3)

A portfolio has five positions of $2 million each. The standard deviation of the returns is 30% for each position. The correlations between each pair of returns is 0.2. **Calculate** the VaR using a Z-value of 2.33.

Answer:

The standard deviation of the portfolio returns is:

$$\sigma_P = 30\%\sqrt{\frac{1}{5} + \left(1 - \frac{1}{5}\right)0.2}$$

$$\sigma_P = 30\%\sqrt{0.36}$$

$$\sigma_P = 18\%$$

The VaR in nominal terms is:

$$VaR_p = Z_c \times \sigma_p \times V = (2.33)(18\%)(\$10 \text{ million})$$

$$VaR_p = \$4{,}194{,}000$$

Portfolio VaR

- **Marginal VAR (边际VAR)**
  - $$\text{Marginal VAR}=\text{MVAR}_i=\frac{\partial \text{VAR}}{\partial(\text{monetary investment in i})}=Z_c\times\frac{\text{cov}(R_i,R_P)}{\sigma_P}$$
  - $$\text{Marginal VAR}=\text{MVAR}_i=\frac{\text{VAR}_P}{P}\times\beta_i$$
- **Incremental VAR (增量VAR)**
  - Is the change in VAR from the addition of a new position in a portfolio
  - Incremental VAR=new VAR from the revaluation - VAR before the addition.
  - Approximate computation: Incremental VAR $\approx \text{MVAR}_i \times P_i$

Example

Example: Computing marginal VaR

Assume Portfolio X has a VaR of €400,000. The portfolio is made up of four assets: Asset A, Asset B, Asset C, and Asset D. These assets are equally weighted within the portfolio and are each valued at €1,000,000. Asset A has a beta of 1.2. **Calculate** the marginal VaR of Asset A.

Answer

$$\text{Marginal VaR}_A = (\text{VaR}_P \,/\, \text{portfolio value}) \times \beta_A$$

$$\text{Marginal VaR}_A = (400{,}000 \,/\, 4{,}000{,}000) \times 1.2 = 0.12$$

Thus, portfolio VaR will change by 0.12 for each euro change in Asset A.

Portfolio VaR

Component VaR

成分VaR

Component VaR for position *i*, denoted $CVaR_i$, is the amount of risk a particular fund contributes to a portfolio of funds. It will generally be less than the VaR of the fund by itself (i.e., stand alone VaR) because of diversification benefits at the portfolio level. In a large portfolio with many positions, the approximation is simply the marginal VaR multiplied by the dollar weight in position *i*:

$$CVaR_i = (MVaR_i) \times (w_i \times P) = VaR \times \beta_i \times w_i$$

Using $CVaR_i$, we can express the total VaR of the portfolio as:

$$VaR = \sum_{i=1}^{N} CVaR_i = VaR\left(\sum_{i=1}^{N} w_i \times \beta_i\right)$$

Given the way the betas were computed we know:

$$\left(\sum_{i=1}^{N} w_i \times \beta_i\right) = 1$$

真题回顾

- Suppose a portfolio consists of a USD 1 million investment in Euros and a USD 4 million investment in Mexican Pesos. Additional information is given below:
  - Portfolio beta of Euro = 0.90
  - Portfolio beta of Peso = 1.30
  - Diversified Portfolio VaR = USD 324,700

  Based on the given information, the marginal VaR and the component VaR of the Euro position are closest to:

	Marginal VaR	Component VaR
A.	USD 0.058	USD 58,446
B.	USD 0.292	USD 292,230
C.	USD 0.084	USD 337,688
D.	USD 0.106	USD 422,110

$$MVAR_{EUR} = \frac{VAR_P}{P} \times \beta_i = \frac{324,700}{5,000,000} \times 0.9 = 0.058446$$

真题回顾

- **EXAMPLE 29.11: FRM EXAM 2009-QUESTION 8-9**

A risk manager assumes that the joint distribution of returns is multivariate normal and calculates the following risk measures for a two-asset portfolio:

Asset	Position	Individual VAR	Marginal VAR	VAR Contribution
1	USD 100	USD 23.3	0.176	USD 17.6
2	USD 100	USD 46.6	0.440	USD 44.0
Total	USD 200	USD 61.6		USD 61.6

If asset 2 is dropped from the portfolio, what is the reduction in portfolio VAR?

A. USD 15.0

B. USD 38.3

C. USD 44.0

D. USD 46.6

Example 29.11: FRM Exam 2009—Question 8-9

b. This is 61.6 minus the portfolio VAR of asset 1 alone, which is USD 23.3, for a difference of 38.3.

Managing Portfolio risk Using VAR

- A manager can *lower a portfolio VAR by lowering allocations to the positions with the highest marginal VAR.*
- Portfolio risk will be at a global minimum where all the marginal VARs are equal for all *i* and *j*:
- $MVAR_i=MVAR_j$

Obtaining Optimal Portfolio Using VAR

- The optimal portfolio has the highest Sharpe ratio:
  - ✓ Sharpe ratio=(portfolio return – risk free rate)/(standard deviation of portfolio return)
- Modifying this formula to the excess return over VAR:
  - ✓ (portfolio return – risk free rate)/(VAR of portfolio)

In order to make this ratio maximized, we have:

$$\frac{\text{Position i return - risk free rate}}{MVAR_i} = \frac{\text{Position j return - risk free rate}}{MVAR_j}$$

For all position i and j.

or

$$\frac{\text{Position i return - risk free rate}}{\beta_i} = \frac{\text{Position j return - risk free rate}}{\beta_j}$$

Example

- A portfolio has an equal amount invested in two positions, X and Y. The expected excess return of X is 9% and that of Y is 12%. Their marginal VARs are 0.06 and 0.075 respectively. To move toward the optimal portfolio, what the manager would probably do:

  A. increase the allocation in Y and/or lower that in X.

  B. increase the allocation in X and/or lower that in Y.

  C. do nothing because the information is insufficient.

  D. not change the portfolio because it is already optimal.

Answer : A

The excess return divided by marginal VAR ratios for X and Y are 1.5 and 1.6, respectively. Therefore, the portfolio weight in Y should increase to move the portfolio toward the optimal portfolio.

- **EXAMPLE 29.13: FRM EXAM 2009-QUESTION 8-12**

  The pension management analysts at Bing Inc. use a two-step process to manage the assets and risk in the pension portfolio. First, they use a VAR-based risk budgeting process to determine the asset allocation across for broad asset classes. Then, within each asset class, they set a maximum tracking error allowance from a benchmark index and determine an active risk budget to distribute among individual managers. Assume the returns are normally distributed. From the first step in the process, the following information is available.

	Expected Return (%)	Volatility (%)	Asset Allocation (%)	Individual VAR (USD)	Marginal VAR
Small cap	0.20%	2.66%	35.0%	6,491	0.055
Large cap	0.15%	2.33%	40.0%	6,497	0.044
Commodities	0.10%	1.91%	16.7%	2,216	0.020
Emerging markets	0.15%	2.70%	8.3%	1,570	0.047
			Total VAR:	13,322	

真题回顾

- Which of the following statements is/are correct?

I. Using VAR as the risk budgeting measure, the emerging markets class has the smallest risk budget.

II. If an additional dollar were added to the portfolio, the marginal impact on portfolio VAR would be greatest if it were invested in small caps.

III. As the maximum tracking error allowance is lowered, the individual managers have more freedom to achieve greater excess returns.

IV. Setting well-defined risk limits and closely monitoring risk levels guarantee that risk limits will not be exceeded.

A. I and II only

B. I,II,III and IV

C. II and III

D. I only

201405真题讲解

7. You are asked to evaluate the VaR of a portfolio of two stocks, A and B, estimated at the 95% confidence level and gather the information in the following table:

Stock	Current Position (USD)	Individual VaR (USD)	Marginal VaR	Beta
A	2,000,000	263,177	0.068	1.3
B	3,000,000	444,110	0.080	0.9
Total	5,000,000			

What is the difference between the undiversified VaR and diversified VaR of this portfolio?

A. USD 314,487
B. USD 331,287
C. USD 353,550
D. USD 376,000

$$VaR_P = V_A \times MVaR_A + V_B \times MVaR_B = 2,000,000 \times 0.068 + 3,000,000 \times 0.080 = 376,000$$

$$(263,177 + 444,110) - 376,000 = 331,287$$

恭祝大家

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顺利通过考试！